Quasi-Static Analysis of a Microstrip Via Through a Hole in a Ground Plane

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Abstract —**The equivalent circuit of a via which connects two semi-infinitely long transmission lines through a circular hole in a ground plane is considered. The π-type equivalent circuit consists of two excess capacitances and an excess inductance. They are quasi-static quantities and thus are computed statically by the method of moments from the integral equations. The integral equations are established by introducing a sheet of magnetic current in the electrostatic case and a layer of magnetic charge in the magnetostatic case. Parametric plots of the excess capacitances, the excess inductance, and the characteristic admittance of the via are given for reference.**

I. INTRODUCTION

THE GEOMETRY of the problem to be considered in this paper is shown in Fig. 1. Two semi-infinitely long transmission lines, wire 1 and wire 2, are connected by a via through a hole in a conducting ground plane. The via consists of wire 3 and wire 4. The radii of wires 1, 2, 3, 4, and the hole, denoted a_1, a_2, a_3, a_3, and a_5, respectively, are very small compared to the heights, h_1 and h_2, of wire 1 and wire 2 with respect to ground. The media in the upper region (region a) and the lower region (region b) may be different. Let us assume that (ϵ_1, μ_1) and (ϵ_2, μ_2) are the constitutive constants for region a and region b, respectively. Also, the media and all the conductors are perfect (lossless). For simplicity, the equivalent circuit of the via is assumed to be π-type, as is shown in Fig. 2. In Fig. 2, Y_{01} is the characteristic admittance of wire 1 above the ground plane and Y_{02} is the characteristic admittance of wire 2 below the ground plane. The circuit of Fig. 2 is valid when only a small portion of the line voltage is dropped across L_e and when only a small portion of the line current is shunted through C_{e1} and C_{e2}. We desire to determine the capacitances C_{e1} and C_{e2} and the inductance L_e. The problem described here is of practical interest. For example, printed circuits on different sides of a ground plane inside computers are often connected by a via through a hole in the ground plane. The related problems of the connection of two perpendicular strips above a ground plane [1], the connection of two parallel wires above a ground plane [2], and the connection of two parallel strips above a ground plane [3], [4] were previously considered.

Manuscript received August 21, 1987; revised January 4, 1988. This work was supported by the Office of Naval Research, Arlington, VA 22217, under Contract N00014-88-K-0027 and by the New York State Center for Advanced Technology in Computer Applications and Software Engineering, Syracuse University.
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IEEE Log Number 8820438.

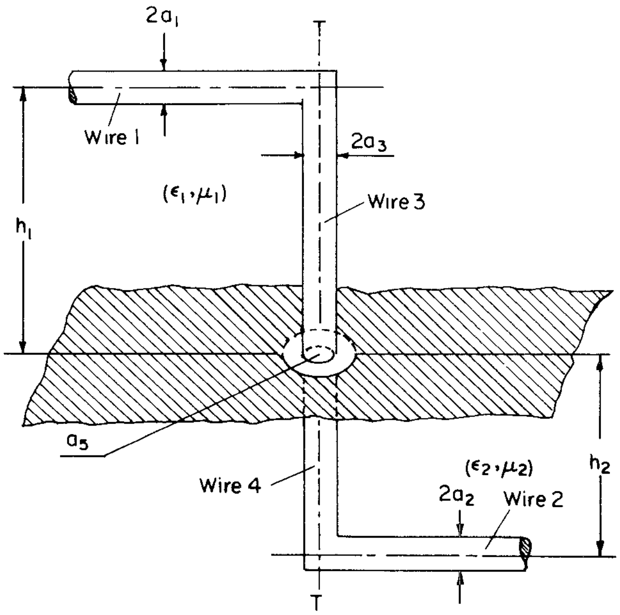

Fig. 1. The geometrical structure of the problem.

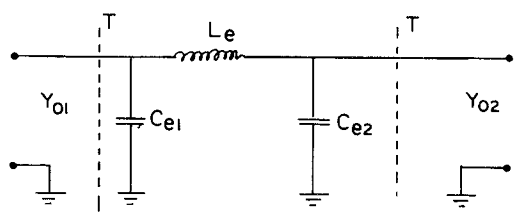

Fig 2 The equivalent circuit for the problem.

The equivalent capacitance C_{e1} of the portion of the via above the ground plane is a quasi-electrostatic quantity and is defined as [2], [3]

$$C_{e1} = \lim_{l_1 \to \infty} \frac{Q_1 + Q_3 - l_1 q_{01}}{V}. \tag{1}$$

Here Q_1 is the total electric charge on the portion of wire 1 of length l_1, Q_3 is the total charge on wire 3, V is the constant voltage maintained at the surface of the wires with respect to ground, and q_{01} is the uniform charge density on wire 1 far away from the via, or equivalently, the charge density required to raise the potential of wire 1 to V volts if the hole were closed, wire 3 were removed, and wire 1 were extended to infinity. Henceforth in the electrostatic case, charge density means charge per unit length. However, since the numerator of the right-hand side of (1) approaches a constant that is very small compared to Q_1 and $l_1 q_{01}$, as l_1 becomes large, numerical calculation of C_{e1} from (1) would result in significant error. To avoid this, we subtract the uniform charge density from the total charge density so that the difference, called the excess charge density, is the unknown in the boundary integral equations. The equivalent capacitance C_{e1} is then the sum of the total excess charge on wire 1 and wire 3 if the voltage V is set to one volt. Hence C_{e1} is also called the excess capacitance of the upper part of the via (wire 3). In a similar manner, the equivalent capacitance C_{e2} is defined and is called the excess capacitance of the lower part of the via (wire 4).

The equivalent inductance L_e is a quasi-magnetostatic quantity defined by [2], [4]

$$L_e = \frac{1}{I}\left\{\int_{\text{wire 1}} (\boldsymbol{A}_1 - \boldsymbol{A}_{10})\cdot d\boldsymbol{l} + \int_{\text{wire 3}} \boldsymbol{A}_3 \cdot d\boldsymbol{l}\right\} + \frac{1}{I}\left\{\int_{\text{wire 2}} (\boldsymbol{A}_2 - \boldsymbol{A}_{20})\cdot d\boldsymbol{l} + \int_{\text{wire 4}} \boldsymbol{A}_4 \cdot d\boldsymbol{l}\right\} \tag{2}$$

where $\boldsymbol{A}_i$ is the total magnetic vector potential on wire i, $i = 1,2,3,4$, due to the steady electric current of filamentary strength I flowing from wire 1 through the via to wire 2. $\boldsymbol{A}_{10}$ is the uniform magnetic vector potential on wire 1 far away from the via or, equivalently, the magnetic vector potential that would be produced by an I ampere current on wire 1 if the hole were closed, wire 3 were removed, and wire 1 were extended to infinity. $\boldsymbol{A}_{20}$ is similarly defined. The sum of the four integrals in (2) is a line integral from a point far to the left on the surface of wire 1 to a point far to the right on the surface of wire 2. Equation (2) is derivable from [4, eq. (1)]. L_e is also called the excess inductance of the via. For convenience, we call the first bracketed term the excess inductance of the upper via (wire 3) and the second bracketed term the excess inductance of the lower via (wire 4).

II. Formulation

The excess capacitances C_{e1} and C_{e2} are quasi-electrostatic quantities and thus computed in the electrostatic case. As shown in Fig. 3, we first close the hole by a conductor and place a sheet of magnetic current $\boldsymbol{M}$ just above the hole and $-\boldsymbol{M}$ just below it. Steady in that it has no surface divergence, this magnetic current is related to the electric field $\boldsymbol{E}_A$ over the hole by

$$\boldsymbol{M} = \boldsymbol{E}_A \times \boldsymbol{n} \tag{3}$$

where $\boldsymbol{n}$ is the unit vector normal to the ground plane and pointing upwards (from region b to region a). By the uniqueness theorem [5, sec. 3-3] and the equivalence principle [5, sec. 3-5], the field remains the same in region a

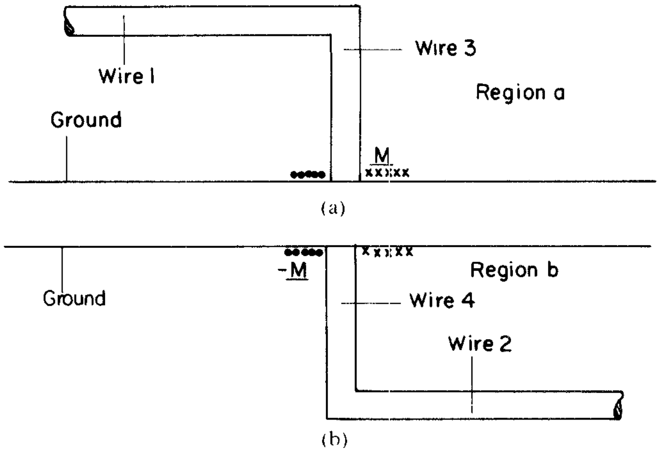

Fig. 3. The electrostatic problem divided into two parts. (a) The electric field remains unchanged in region a if the hole is closed and $\boldsymbol{M}$ is placed above the hole. (b) The electric field remains unchanged in region b if the hole is closed and $-\boldsymbol{M}$ is placed just below the hole.

and region b if

$$\epsilon_1 \frac{\partial \phi^{\mathrm{a}}}{\partial n} = \epsilon_2 \frac{\partial \phi^{\mathrm{b}}}{\partial n} \qquad \text{in the hole} \tag{4}$$

where ϕ^i is the electric potential in region i, $i = \mathrm{a,b}$. In addition to (4) the following boundary conditions must also be satisfied:

$$\begin{aligned} \phi^{\mathrm{a}} &= V \qquad \text{on wires 1 and 3} \\ \phi^{\mathrm{b}} &= V \qquad \text{on wires 2 and 4.} \end{aligned} \tag{5}$$

Neither wire 3 nor wire 4 is connected to the ground plane in Fig. 3.

The total charge density may be recognized as the sum of the excess charge density and uniform charge density, i.e.,

$$q = \begin{cases} q_{e1} + q_{01} & \text{on wire 1} \\ q_{e3} & \text{on wire 3} \\ q_{e4} & \text{on wire 4} \\ q_{e2} + q_{02} & \text{on wire 2} \end{cases} \tag{6}$$

where the subscript e on q denotes excess charge density. q_{01} and q_{02} are known and are given by

$$\begin{aligned} q_{01} &= \frac{2\pi\epsilon_1 V}{\ln(2h_1/a_1)}, \qquad h_1 \gg a_1 \\ q_{02} &= \frac{2\pi\epsilon_2 V}{\ln(2h_2/a_2)}, \qquad h_2 \gg a_2. \end{aligned} \tag{7}$$

If the potential V is set to one volt, the total excess charge will give rise to the desired excess capacitance, i.e.,

$$\begin{aligned} C_{e1} &= \int_{\text{wire 1}} q_{e1}\, dl + \int_{\text{wire 3}} q_{e3}\, dl \\ C_{e2} &= \int_{\text{wire 2}} q_{e2}\, dl + \int_{\text{wire 4}} q_{e4}\, dl. \end{aligned} \tag{8}$$

Let $\phi(q_{ej})$ denote the potential due to q_{ej} in the presence of the completed ground plane (hole closed), $\phi^+(\boldsymbol{M})$ the potential in region a due to $\boldsymbol{M}$ residing on the region a side of the completed ground plane, and $\phi^-(M)$ the potential in region b due to M residing on the region b side. Then,

$$\phi^a = \phi(q_{01}) + \phi(q_{e1}) + \phi(q_{e3}) + \phi^+(M)$$
$$\phi^b = \phi(q_{02}) + \phi(q_{e2}) + \phi(q_{e4}) - \phi^-(M). \quad (9)$$

Note that the electric potential due to a sheet of steady magnetic current is analogous to the magnetic scalar potential due to a sheet of steady electric current. The latter is studied in many fundamental electromagnetic field theory books, for example, [6]. Substitution of (9) into (4) and (5), with V being set to one volt, yields integral equations for the excess charge densities:

$$\phi(q_{e1}) + \phi(q_{e3}) + \phi^+(M) = 1 - \phi(q_{01}) \quad \text{on wires 1 and 3}$$
$$\phi(q_{e2}) + \phi(q_{e4}) - \phi^-(M) = 1 - \phi(q_{02}) \quad \text{on wires 2 and 4}$$
$$\epsilon_1 \frac{\partial \phi(q_{e1})}{\partial n} + \epsilon_1 \frac{\partial \phi(q_{e3})}{\partial n} + \epsilon_1 \frac{\partial \phi^+(M)}{\partial n} - \epsilon_2 \frac{\partial \phi(q_{e2})}{\partial n} - \epsilon_2 \frac{\partial \phi(q_{e4})}{\partial n} + \epsilon_2 \frac{\partial \phi^-(M)}{\partial n} = -\epsilon_1 \frac{\partial \phi(q_{01})}{\partial n} + \epsilon_2 \frac{\partial \phi(q_{02})}{\partial n} \quad \text{in the hole.} \quad (10)$$

Although q_{e1} and q_{e2} exist on the semi-infinitely long wires 1 and 2, they decay to zero rapidly as one moves away from the via. We may truncate q_{e1} and q_{e2} and the boundary equations on wire 1 and wire 2 at some distances, say $3h_1$ and $3h_2$, from the via. By doing so, we neglect the contribution to the excess capacitances from the excess charge beyond the lengths $3h_1$ on wire 1 and $3h_2$ on wire 2. Now, the method of moments may be used to solve for q_{e1}, q_{e2}, q_{e3}, q_{e4}, and M numerically. We divide wire 1 and wire 2 (truncated) and wire 3 and wire 4 into subsections, assume uniform charge distribution on each subsection, and enforce the first two of equations (10) at the center of each subsection. Furthermore, we divide the hole into annuluses, assume uniform circulating current distribution on each annulus, and enforce the third of equations (10), which is averaged over the interval from 0 to 2π for the azimuthal variation, at the midpoint between the edges of each annulus. A detailed discussion of the moment method as applied to this problem is presented in [7].

Now, we turn to the magnetostatic case to compute the excess inductance. Similar to the electrostatic case, we close the hole by a conductor and place a layer of magnetic charge density m just above the hole and $-m$ just below it, as is shown in Fig. 4. This m is equal to the normal magnetic field over the hole. Again by the uniqueness theorem and the equivalence principle, the magnetic field remains unchanged in region a and region b if

$$H^a_{tan} = H^b_{tan} \quad (11)$$

is enforced, where H^i_{tan} is the tangential magnetic intensity

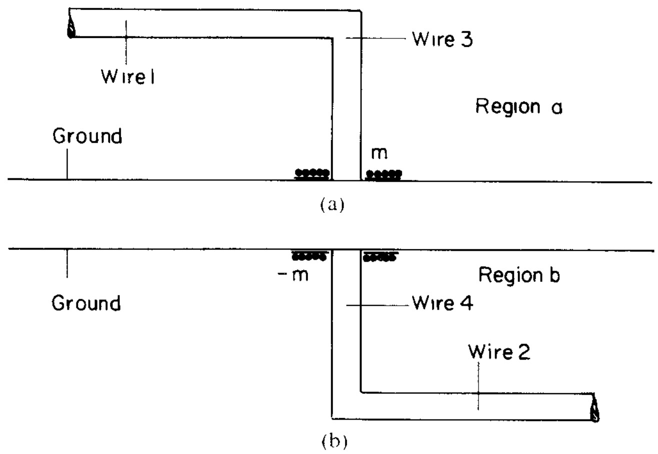

Fig. 4. The magnetostatic problem divided into two parts. (a) The magnetic field remains unchanged in region a if the hole is closed and m is placed just above the hole. (b) The magnetic field remains unchanged in region b if the hole is closed and $-m$ is placed just below the hole.

over the hole in region i, $i =$ a, b. Let H (wire i) denote the magnetic intensity due to the electric current on wire i in the presence of the completed ground plane for $i = 1,2,3,4$, $H^+(m)$ the magnetic intensity due to m above the completed ground plane, and $H^-(m)$ the magnetic intensity due to m below the completed ground plane. We can write

$$H^a = H(\text{wire } 1) + H(\text{wire } 3) + H^+(m)$$
$$H^b = H(\text{wire } 2) + H(\text{wire } 4) - H^-(m). \quad (12)$$

Note that the magnetic intensity due to a layer of magnetic charge is analogous to the electric field due to a layer of electric charge, as discussed in [6]. Therefore, $H^+(m)$ and $H^-(m)$ may be represented by the gradients of some scalar functions $\Psi^+(m)$ and $\Psi^-(m)$. That is,

$$H^+(m) = -\nabla\Psi^+(m)$$
$$H^-(m) = -\nabla\Psi^-(m). \quad (13)$$

Substitution of (12) and (13) into (11) gives

$$\nabla\Psi^+(m) + \nabla\Psi^-(m)|_{tan} = H(\text{wire } 1) - H(\text{wire } 2) + H(\text{wire } 3) - H(\text{wire } 4)|_{tan}. \quad (14)$$

Equation (14), an integral equation for m, implies that $\Psi^+(m) + \Psi^-(m)$ equals Ψ^{inc}, where Ψ^{inc} is a potential whose gradient is the right-hand side of (14). To compensate for the fact that Ψ^{inc} is only known to within an additive constant, we require that the total magnetic charge associated with m vanish. The moment method may be applied now to solve the scalar equation derived from (14) subject to the above constraint on m.

Since there is no coupling between wire 1 and wire 3 and between wire 2 and wire 4, we have

$$A_1 = A_1(\text{wire } 1) + A_1^+(m)$$
$$A_2 = A_2(\text{wire } 2) - A_2^-(m)$$
$$A_3 = A_3(\text{wire } 3) + A_3^+(m)$$
$$A_4 = A_4(\text{wire } 4) - A_4^-(m) \quad (15)$$

where A_i(wire i) is the magnetic vector potential on the surface of wire i due to the current I on wire i in the presence of the completed ground plane, $i=1,2,3,4$. Moreover, $A_i^+(m)$ is the vector potential on the surface of wire i, $i=1,3$, due to m residing on the region a side of the completed ground plane, and $A_i^-(m)$ is the vector potential on the surface of wire i, $i=2,4$, due to m residing on the region b side. Note that (15) is valid only for the vector potential component tangent to each wire and that the other component is not of interest. Letting I be one ampere and putting (15) into (2), we get the excess inductance of wire 3:

$$L_{e1}=\int_{\text{wire 1}}(A_1(\text{wire 1})-A_{10})\cdot dl+\int_{\text{wire 3}}A_3(\text{wire 3})\cdot dl+\int_{\text{wire 1}}A_1^+(m)\cdot dl+\int_{\text{wire 3}}A_3^+(m)\cdot dl. \tag{16}$$

The excess inductance of wire 4 is obtained from the above expression by replacing all the 1's by 2's, 3's by 4's, m's by $-m$'s, and superscripts + by −, that is,

$$L_{e2}=\int_{\text{wire 2}}(A_2(\text{wire 2})-A_{20})\cdot dl+\int_{\text{wire 4}}A_4(\text{wire 4})\cdot dl-\int_{\text{wire 2}}A_2^-(m)\cdot dl-\int_{\text{wire 4}}A_4^-(m)\cdot dl. \tag{17}$$

Because of the small radii of the wires, the current on the surface of each wire may be approximated by a filamentary current I on the axis of the wire. Thus the first two integrals in (16) can be evaluated analytically [2]. The last two integrals may be expressed in terms of magnetic intensities according to Stokes's theorem. The result is

$$L_{e1}=\frac{\mu_1}{4\pi}\left[2h_1\ln\left(\frac{4h_1}{a_3}\right)-4h_1+a_1+a_3\right]+\mu_1\int_{S_1}H_n^+(m)\,ds \tag{18}$$

where in the last integral, S_1 is the planar surface bounded by the axes of wire 1 and wire 3 and the ground. The subscript n on $H^+(m)$ denotes the component normal to S_1 and pointing into the paper. The expression for L_{e2} is similar to (18) in form. The excess inductance of the via is then the sum of L_{e1} and L_{e2}.

III. Numerical Results and Discussion

In order to implement the computer program on a PC/AT, we simplify the computation by choosing only one expansion function for the magnetic current and none for the magnetic charge. This is justified as follows. Since the radius of the hole is very small compared to the heights h_1 and h_2, the couplings (electric and magnetic) through the hole between wire 1 and wire 2 are negligible to the first-order approximation. In other words, the fields in the hole are primarily from the via (wire 3 and wire 4). In the electrostatic case, the tangential electric field in the hole is in the radial direction with the variation being of the form

$$E_A=\frac{K}{\rho\sqrt{1-(\rho/a_5)^2}} \tag{19}$$

where ρ is the radial distance from the center of the hole and K is a constant determined by

$$\int E_A\cdot dl=1. \tag{20}$$

The integration path is chosen in the radial direction from the surface of wire 3 to the edge of the hole. Substitution of (19) into (20) gives

$$K=\frac{1}{\ln\left(a_5/a_3+\sqrt{(a_5/a_3)^2-1}\right)}. \tag{21}$$

Thus the magnetic current is circulating and its amplitude is given by

$$M=\frac{1}{\ln\left(a_5/a_3+\sqrt{(a_5/a_3)^2-1}\right)}\cdot\frac{1}{\rho\sqrt{1-(\rho/a_5)^2}}. \tag{22}$$

In the magnetostatic case, the normal magnetic field in the hole is negligible. Hence the magnetic charge may be neglected and a closed form for the excess inductance of the via is obtained. That is,

$$L_e=\frac{\mu_1}{2\pi}h_1\ln\left(\kappa_1\frac{h_1}{a_3}\right)+\frac{\mu_2}{2\pi}h_2\ln\left(\kappa_2\frac{h_2}{a_3}\right) \tag{23}$$

where κ_1 and κ_2 are constants. Approximately, $\kappa_1=\kappa_2=0.5413$.

In Figs. 5–7, the normalized excess capacitance of the upper via (wire 3) $C_{e1}/(\epsilon_1 a_3)$ is plotted. The curves are also applicable to the normalized excess capacitance $C_{e2}/(\epsilon_2 a_3)$ of the lower via (wire 4) if all the subscripts 1 are replaced by 2. In Figs. 8 and 9[1] the normalized excess inductance $L_e/(2\mu_1 a_3)$ of a symmetric via is plotted. A via is symmetric if $(h_1, a_1, \epsilon_1, \mu_1)=(h_2, a_2, \epsilon_2, \mu_2)$. These plots can also be viewed as plots for the normalized excess inductance $L_{e1}/(\mu_1 a_3)$ of the upper via. If the subscripts 1 are replaced by 2, they become plots of $L_{e2}/(\mu_2 a_3)$, the normalized inductance of the lower via. In Figs. 10 and 11 the characteristic admittance $\eta_1 Y_e$ of a symmetric via is plotted. Here η_1 is the intrinsic impedance of the medium. The characteristic admittance of the via is defined as

$$Y_e=\sqrt{(C_{e1}+C_{e2})/L_e}. \tag{24}$$

If a TEM wave approaches the via along wire 1 and if wire 2 is terminated with a matched load (Y_{02}), then it can be shown that the least reflection will occur when

$$Y_e=\sqrt{Y_{01}Y_{02}}. \tag{25}$$

[1] Note that the curves in Figs. 8 and 9 are plotted under the assumption that the hole is small in relation to the heights h_1 and h_2. To emphasize this and to be consistent with other plots, we add the restriction $a_5=2a_3$.

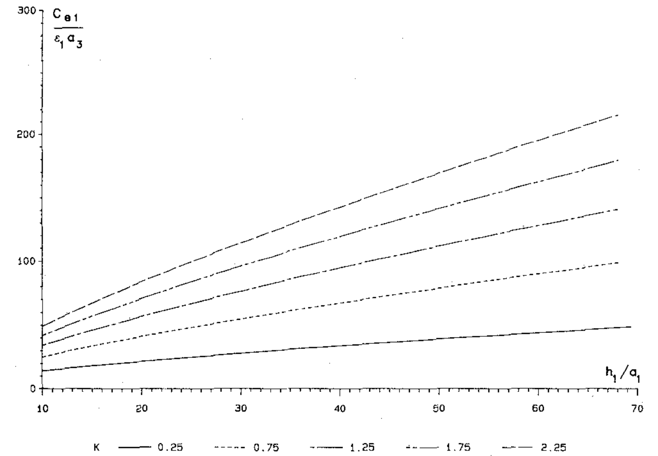

Fig. 5. Normalized excess capacitance of the upper via ($a_5 = 2a_3$, $k = a_1/a_3$).

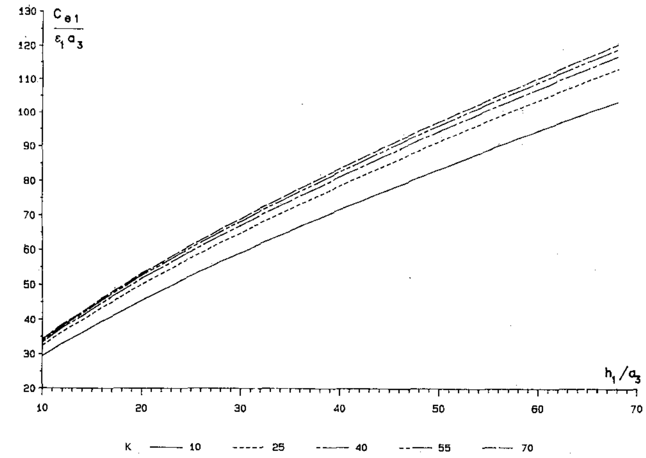

Fig. 7. Normalized excess capacitance of the upper via ($a_5 = 2a_3$, $k = h_1/a_1$).

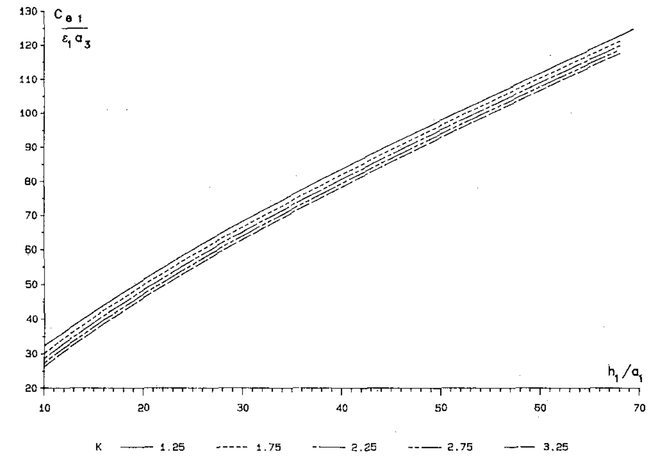

Fig. 6. Normalized excess capacitance of the upper via ($a_1 = a_3$, $k = a_5/a_3$).

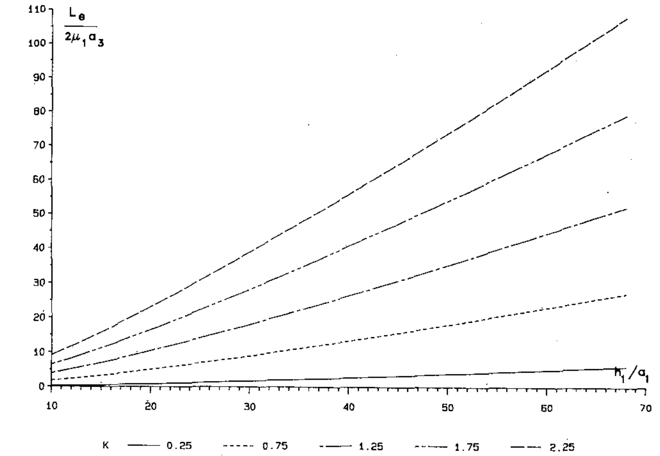

Fig. 8. Normalized excess inductance of a symmetric via ($a_5 = 2a_3$, $k = a_1/a_3$).

To see this, we notice that the voltage reflection coefficient at the T plane on wire 1 in Fig. 2 is given by

$$\Gamma = \frac{\dfrac{Y_{01}\left(1-\omega^2 L_e C_{e2}\right) - Y_{02}\left(1-\omega^2 L_e C_{e1}\right)}{j\omega L_e} + Y_{01}Y_{02} - \dfrac{1}{L_e}\left[C_{e1}\left(1-\dfrac{\omega^2 L_e C_{e2}}{2}\right) + C_{e2}\left(1-\dfrac{\omega^2 L_e C_{e1}}{2}\right)\right]}{\dfrac{Y_{01}\left(1-\omega^2 L_e C_{e2}\right) + Y_{02}\left(1-\omega^2 L_e C_{e1}\right)}{j\omega L_e} + Y_{01}Y_{02} + \dfrac{1}{L_e}\left[C_{e1}\left(1-\dfrac{\omega^2 L_e C_{e2}}{2}\right) + C_{e2}\left(1-\dfrac{\omega^2 L_e C_{e1}}{2}\right)\right]} \tag{26}$$

where ω is the angular frequency. Usually, $\omega^2 L_e(C_{e1} + C_{e2}) \ll 1$. The above expression then reduces to

$$\Gamma = \frac{Y_{01} - Y_{02} + j\omega L_e Y_{01} Y_{02} - j\omega\left(C_{e1} + C_{e2}\right)}{Y_{01} + Y_{02} + j\omega L_e Y_{01} Y_{02} + j\omega\left(C_{e1} + C_{e2}\right)}. \tag{27}$$

When (25) is satisfied, the magnitude of the numerator of (27) assumes its minimum, the magnitude of the denominator is roughly $Y_{01} + Y_{02}$, and thus $|\Gamma|$ is minimized. The reflection from the via is minimized. Furthermore, if (25) is satisfied and if the system is symmetric ($Y_{01} = Y_{02}$), there is no reflection from the via. That is, all of the power from the incident wave will be transmitted through the via to the matched load. In this case, the via is called reflectionless.

To design a reflectionless via, consider the following numerical example. Suppose that

$$\begin{aligned} h_1 &= h_2 = 1.00 \text{ cm} \\ a_1 &= a_2 = 0.10 \text{ cm} \\ a_5 &= 2a_3 \end{aligned} \tag{28}$$

and that we wish minimize reflection from the via. Since

$$Y_{01} = Y_{02} = \frac{2\pi}{\eta \ln\left(2h_1/a_1\right)} = \frac{2.10}{\eta} \tag{29}$$

the reflectionless condition becomes

$$\eta Y_e = 2.10. \tag{30}$$

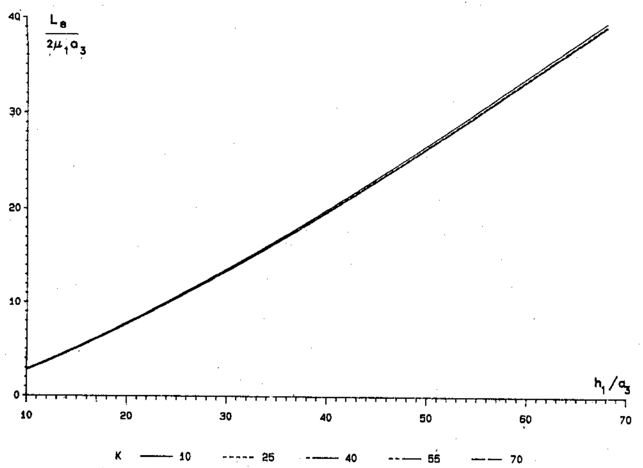

Fig. 9. Normalized excess inductance of a symmetric via ($a_5 = 2a_3$, $k = h_1/a_1$).

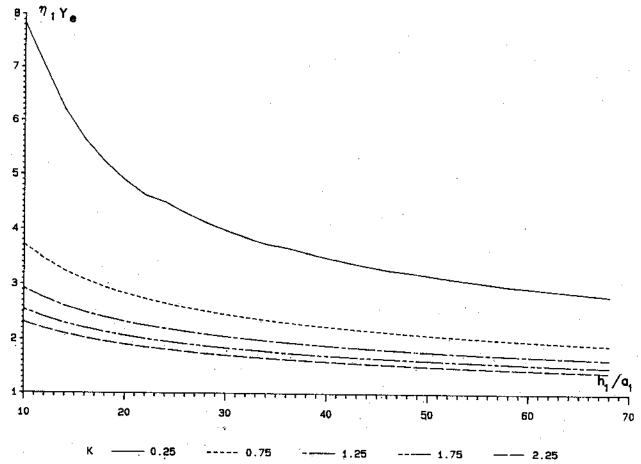

Fig. 10. Normalized characteristic admittance of a symmetric via ($a_5 = 2a_3$, $k = a_1/a_3$).

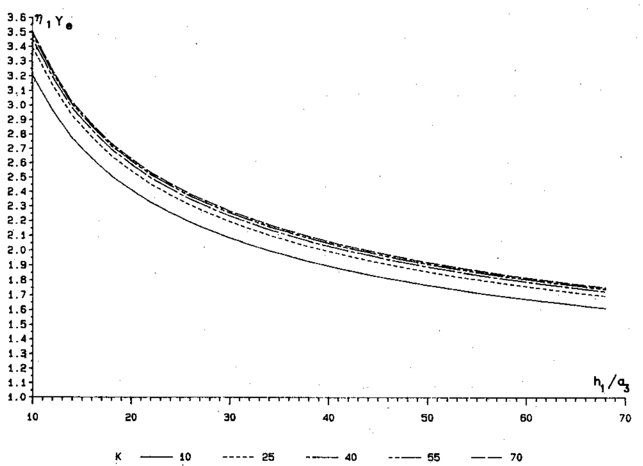

Fig. 11. Normalized characteristic admittance of a symmetric via ($a_5 = 2a_3$, $k = h_1/a_1$).

Using the $k = 10$ curve in Fig. 11, we find that the above condition is satisfied when $a_3 = 0.034$ cm. Hence, given (28), the via will be reflectionless when the radius of the via is 0.034 cm.

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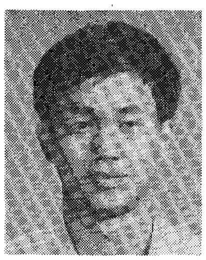

Taoyun Wang (S'84) was born in 1963. He received the B.S. degree in 1982 from Xian Jiaotong University, China, and the M.S. degree in 1984 from Syracuse University, Syracuse, NY, both in electrical engineering. He is currently working toward the Ph.D. degree in electrical engineering and the M.S. degree in mathematics at Syracuse University. Since September 1983, he has been actively involved with the electromagnetic research group at Syracuse University. His research areas include electromagnetic scattering and transmission, microstrip circuits, and numerical methods.

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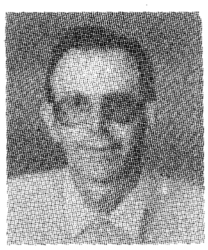
From 1945 to 1946, he served as an Instructor at the U.S. Naval Radio Materiel School, Dearborn, MI, and from 1948 to 1950, he was employed as an Instructor and Research Assistant at Syracuse University. While studying at Ohio State University, he served as a Research Fellow in the Antenna Laboratory. Since 1952, he has been on the faculty of Syracuse University, where he is presently Professor of Electrical Engineering. During 1959–1960, he was visiting Associate Professor at the University of Illinois, Urbana; in 1964 he was Visiting Professor at the University of California, Berkeley; and in 1969 he was Guest Professor at the Technical University of Denmark, Lyngby, Denmark.

Dr. Harrington is a member of Tau Beta Pi, Sigma Xi, and the American Association of University Professors.



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